Comonotonicity of asset prices in arbitrage-free markets

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Abstract

Consider a financial market with a riskless interest rate r and a single underlying asset. Let S_t be the asset price at time t. For times $t_1, t_2, \ldots, t_n$, we investigate the situation where the vector is comonotonic. We prove that when the financial market is arbitrage-free, the vector of asset prices $(S_{t_1}, S_{t_2}, \ldots, S_{t_n})$ is comonotonic and moreover, the r.v. S_{t_1} has no isolated mass, then the following linear relationship must hold between the asset prices at different times:

$$S_{t_j} = S_{t_i}\ e^{r(t_j - t_i)}, \quad i, j = 1, 2, \ldots, n \quad \text{a.s.}$$

We give examples to illustrate that in a multi-asset arbitrage-free market the vector $\left(S_t^1, S_t^2, \ldots, S_t^n\right)$ of the different asset prices at time t may be comonotonic.

Consider the B&S multi-asset market

$$\frac{dS_t^i}{S_t^i} = \mu_i dt + \sum_{j=1}^{d} \sigma_{ij} dW_t^j, \quad i = 1, 2, \ldots, m.$$

Here $\left(W_t^1, W_t^2, \ldots, W_t^n\right)$ is a standard d-dimensional Brownian motion that generates the underlying filtration. Suppose additionally that for each asset the dividends are paid according to the formula

$$dD_t^i = k_i S_t^j dt, \quad i = 1, 2, \ldots, m.$$

where the D_t^i denote the cumulative dividend payments and the k_i are the positive constant dividends rates.

We review criteria for this market to be arbitrage-free and complete. Related problems are studied in [1], pp. 250-251.

Keywords: Arbitrage-free market, Black & Scholes multi-asset market, Comonotonic distribution, Complete market.

Reference: M.S. Joshi, The Concepts and Practice of Mathematical Finance, Cambridge, 2003.

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